XXIII. *On the Friction and Resistance of Fluids.* *By* George Rennie, *Esq. V.P.R.S.*

Read June 16, 1831.

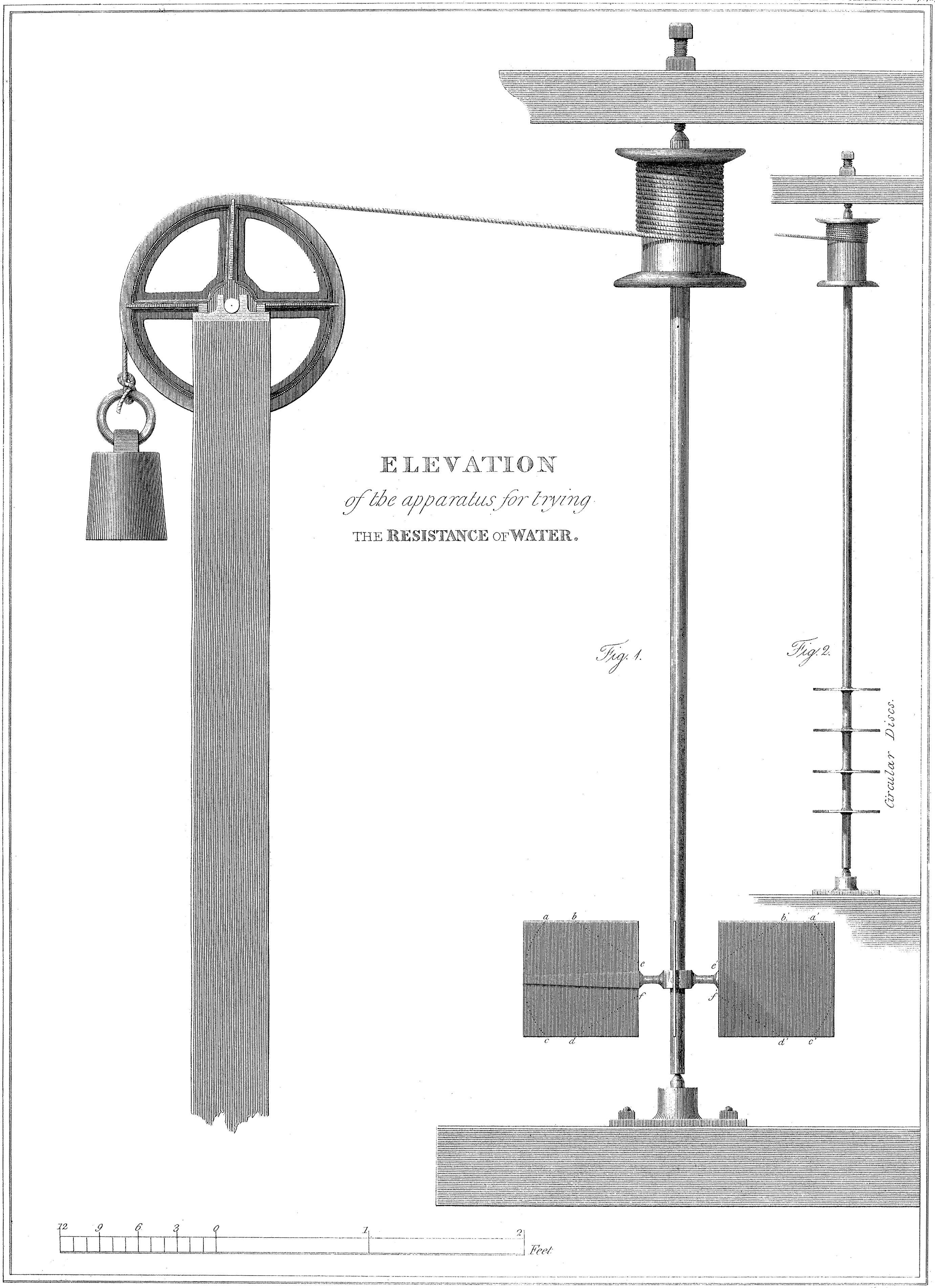
When on a former occasion I communicated the results of a series of experiments on the Friction and Resistance of the Surfaces of Solids (Philosophical Transactions for 1828), I stated that they formed part only of a series of experiments on the nature of friction generally. My object at first was to trace the relation subsisting between the retardation produced by the surfaces of solids in motion when in contact with each other and with fluids; but finding that the subject connected with either of these branches was sufficiently extensive, I deemed it necessary to postpone the second part of the inquiry to a future occasion. Those experiments, however, established some important facts. They showed that (within the limits of abrasion) friction was the same for all solids, and that it was neither affected by surface nor velocity. Subsequent experiments upon rolling bodies of great weight and magnitude, when the resistance was reduced $\frac{1}{1000}$th part of the mass, and the surfaces in the ratio of 13 to 1, have corroborated the affinity of resistance between rolling and sliding bodies. Thus in connecting and continuing the isolated experiments of Coulomb and Vince, and assigning values to the abrasive resistances of most of the most useful solids, a considerable advance has been made in the science.

The subject of the present paper, however, involves difficulties of a more complicated kind. The theory of solids as deduced from the laws of mechanics, and independent of experiment, may be applied to any system of bodies; but the theory of fluids, in which the form and the disposition of the particles, or the laws of their action, are unknown, must necessarily be founded on experiment; and even with this aid, which can only be obtained through the intervention of a solid, our knowledge of the true properties of fluids must be vague and uncertain. Accordingly we find that the subject of fluids attracted the attention of some of the most distinguished mathematicians and philosophers of Europe for the last two centuries; that is, from the year 1628, when CASTELLI first published his Treatise on the Measure of running Water, down to the hydraulic investigations of EYTELWEIN and YOUNG. Between these periods, Italy, France, Germany and England, added their contributions to the science. But it is to the Italians principally that we owe the foundation of it, in their numerous investigations and controversies on the rivers of Italy; hence the writings of CASTELLI, VIVIANI, ZENDRINI, MANFREDI, POLINI, FRISI, GULIELMINI, LECHI, MICHELLOTTI, and of many others *.

Each of them has endeavoured to establish a theory applicable to rivers and torrents, but in general with indifferent success. The science again received fresh accessions from the more valuable investigations of BOSSUT, DUBUAT, VENTURI, FÜNCK, BRUNNING, BIDONE, COULOMB, PRONY, EYTELWEIN and GIRARD; and among our own countrymen, of M'CLAURIN, VINCE, MATTHEW YOUNG, Dr. JURIN, Professor ROBINSON, and the late Dr. THOMAS YOUNG. Sir ISAAC NEWTON had already demonstrated, in his celebrated propositions 51, 52, and 53, of the Principia, (in the case of a cylinder in motion immersed in a fluid,) that the resistance arising from the want of a perfect lubricity in fluids is (cæteris paribus) proportional to the velocity with which the parts of a fluid separated from each other; and that, if a solid cylinder of infinite length revolves with a uniform motion round a fixed axis, in a uniform and infinite fluid, the periodical times of the parts of the fluid thus put in motion will be proportional to their distances from the axis. This theory (although conformable to experiment) was objected to by BERNOULLI and D'ALEMBERT, on the ground that Sir ISAAC NEWTON had not taken into consideration the centrifugal force or friction arising from the pressure of the concentric rings or filaments round the cylinder, the fluid being supposed in a state of permanence, and the friction of the rings equal throughout.

PITOT (1728), in his experiments on the water-works at Marly and Versailles, was the first to demonstrate that with equal velocities, and in the ratio of the volume of water, the friction of water in pipes was in the inverse ratio of their

* Raccotta d' Autori che trattano del Moto dell' Acque.


ELEVATION
of the apparatus for trying
THE RESISTANCE OF WATER.
Fig. 1.
Fig. 2.
Circular Discs.
a b b′ a′
e e′
f f′
c d d′ c′
12 9 6 3 0 1 2
Feet

Js. Basire sc.

diameters; and COUPLET (1733), MARIOTTE, and DEPARCIEUX, estimated the difference between the real and calculated expenditures of glass tubes and pipes.

CHEZY (in 1771 and 1786) was the first engineer who endeavoured to establish the relation subsisting between the inclination of an aqueduct and the transverse section of the volume of water it ought to carry,—on the supposition that the accelerating force, due to the inclination of the bed of the conduit, is counterbalanced by the resistances of the channel in the ratio of the surface, and increasing in proportion to the square of the velocity. What CHEZY had remarked was concluded by BOSSUT, who cleared the investigation of most of its difficulties, and demonstrated it to be in accordance with theory. He found that small orifices discharged less water in proportion than great ones, on account of friction; that the vena contracta, and consequent expenditure, diminished with the height of the reservoir: he pointed out the law by which the discharge diminishes according to the inclination and number of bends in a pipe, and the influence of friction in retarding the velocity of waters moving in canals and pipes, in which he made the square of the velocity to be in the inverse ratio of the length of the pipe: he determined the co-efficients by experiment, and thus obtained a formula expressive of the conditions of the uniform motion of water in open canals. The greater part of these hypotheses may be said to have been removed by the more extensive researches of DUBUAT. His great hydraulic work, published in 1779 and 1786, contains a series of the most valuable observations, whose results accord very nearly with the new formula of the motion of water in pipes and open conduits; and his experiments, with pipes inclined in various angles from the 40,000th part of a right angle to 90 degrees, and in channels which varied from a line and a half in diameter to areas of seven or eight square toises, seem to comprehend every case of inclination; so that by collecting a prodigious number of facts, both with compressible and incompressible fluids, he obtained a general expression for all cases relative to the friction and cohesion of fluids: but a logarithmic function which he introduces in it, by a sort of approximation, gives it a character of uncertainty, which restrains its use, and shows the necessity of fresh researches. VENTURI, in 1798, "Sur la Communication latérale du Mouvements dans les Fluides," repeated and added many new facts to the experiments of BOSSUT, on the expenditure of differently shaped orifices and tubes, but particularly on the lateral communication of motion by the cohesion of fluids. COULOMB first approximated to the solution of the question, by a very ingenious apparatus, consisting of discs of different sizes, fixed by their centres to the lower extremity of a brass wire, and made to oscillate in fluids by the force of torsion only; he concluded that the resistance was a function, composed of two terms, one proportional to the first, the other to the second powers of the resistance: again, that it was not sensibly increased by increasing the height of the fluid, but simply by the cohesion of the particles of the fluid which presented greater or less resistance, in proportion to the viscidity of the fluid, oil being to water in the ratio of 17.5 to 1. But whatever might be the conclusions of COULOMB, it is obvious that both the size and construction of his apparatus were ill calculated to produce results whereon to found a satisfactory theory; and accordingly both Messrs. PRONY and GIRARD, in expressing their formulæ of resistance, have not admitted that of COULOMB, but have adopted the mean of the best of experiments made by other authors: but as these formulæ give only the mean velocity, which is much greater than the velocity (of the fluid contiguous to the pipe) which ought alone to enter into the expression of the retarding force, it follows, that the coefficients deduced from the mean of all the experiments adopted by these gentlemen, have a value greatly inferior to the motion of the fluid contiguous to the side of the pipe or conduit. To ascertain correctly the value of this kind of resistance, M. GIRARD (vide les Mémoires des Sçavans etrangers for 1815), undertook a prodigious number of experiments on tubes of different diameters and length, from which he deduced that the retardation is as the velocity simply. The effects of temperature are very remarkable; if the velocity be expressed by 10, when the temperature is 0° centigrade thermometer, the velocity will be 42°, or increased four times when the temperature is 85°: these values must be deemed approximations only.

The contributions of British philosophers towards the improvement of this science have been, unfortunately, scanty; for, with the exception of Sir ISAAC NEWTON (who led the way), Dr. JURIN, Dr. MATTHEW YOUNG, Dr. DESAGULIERS, Dr. VINCE, Mr. SMEATON, Mr. BANKS, and the late Dr. THOMAS YOUNG, (see the paper of the latter gentleman in the Philosophical Transactions, and his commentaries on EYTELWEIN'S experiments,) we can scarcely find any experiments on the subject*: whatever has been effected by our engineers or scientific men, has either been withheld from the public, or consigned to obscurity; and though we have tracts of marshes and fen land, consisting of many thousand acres, the dissertations on the mode of draining and carrying off their superfluous waters are confined to local pamphlets and reports, of comparatively minor interest to the science of hydraulics.

From the foregoing short but imperfect history, it is obvious that much has been done towards perfecting this science. It is however certain, that much yet remains to be accomplished; and although we are deeply indebted to both the French and English philosophers for their extensive investigations on the laws of capillary attraction, the descents of globes in fluids, and the adhesion of fluids to metal discs, the phenomena of fluidity, and the laws which govern the motion and equilibrium of their particles, must yet remain a problem purely geometrical; and as we possess no tangible means of approximating to the solution of the problem, but through the intervention of a solid, we must content ourselves, in like manner, with the imperfect formulæ deduced from experiments made on a small scale on the friction and adhesion of water in pipes and conduits, until we can ascertain more correctly the causes of the retardations of rivers as they occur in nature.

In the consideration of this question, therefore, I propose to examine, first, the retardations of the surfaces of solids moving in fluids at rest; secondly, the retardations of fluids over solids; and, thirdly, the direct resistance of solids revolving in fluids at rest.

To illustrate the first case, I caused an apparatus to be constructed, of which the annexed Plate XI. is a representation; it consists simply of a cylinder of wood ten inches and three quarters in diameter, and twenty-four inches long, and divided into eight sections of three inches in each, and fixed upon a spindle of iron about four feet in length, and one inch and a quarter thick. The apparatus was accurately turned and polished. Upon the upper part of the spindle, a small cylinder or pulley, six inches in diameter was fixed, and a fine flexible silken cord, communicating with the weight, was wound;

* The experiments of the Society for the Improvement of Naval Architecture, in the years 1793, 1794 1795, 1796, 1797, 1798, relate principally to the resistances of solids moving through fluids.

the apparatus was then fixed in an iron frame, and the frame let into a groove in two upright posts, driven into the bed of the river Thames.

The object of the frame was to allow the cylinder to slide up and down with the level of the tide, and immerse it more or less according to the experiment required to be tried. The friction of the apparatus, or the time that the weight took to descend in the atmosphere, was first noticed; after which it was successively immersed in the water three, six, nine, twelve, fifteen, eighteen, twenty-one, and twenty-four inches, the difference of time showing the retardation according to the annexed Table.

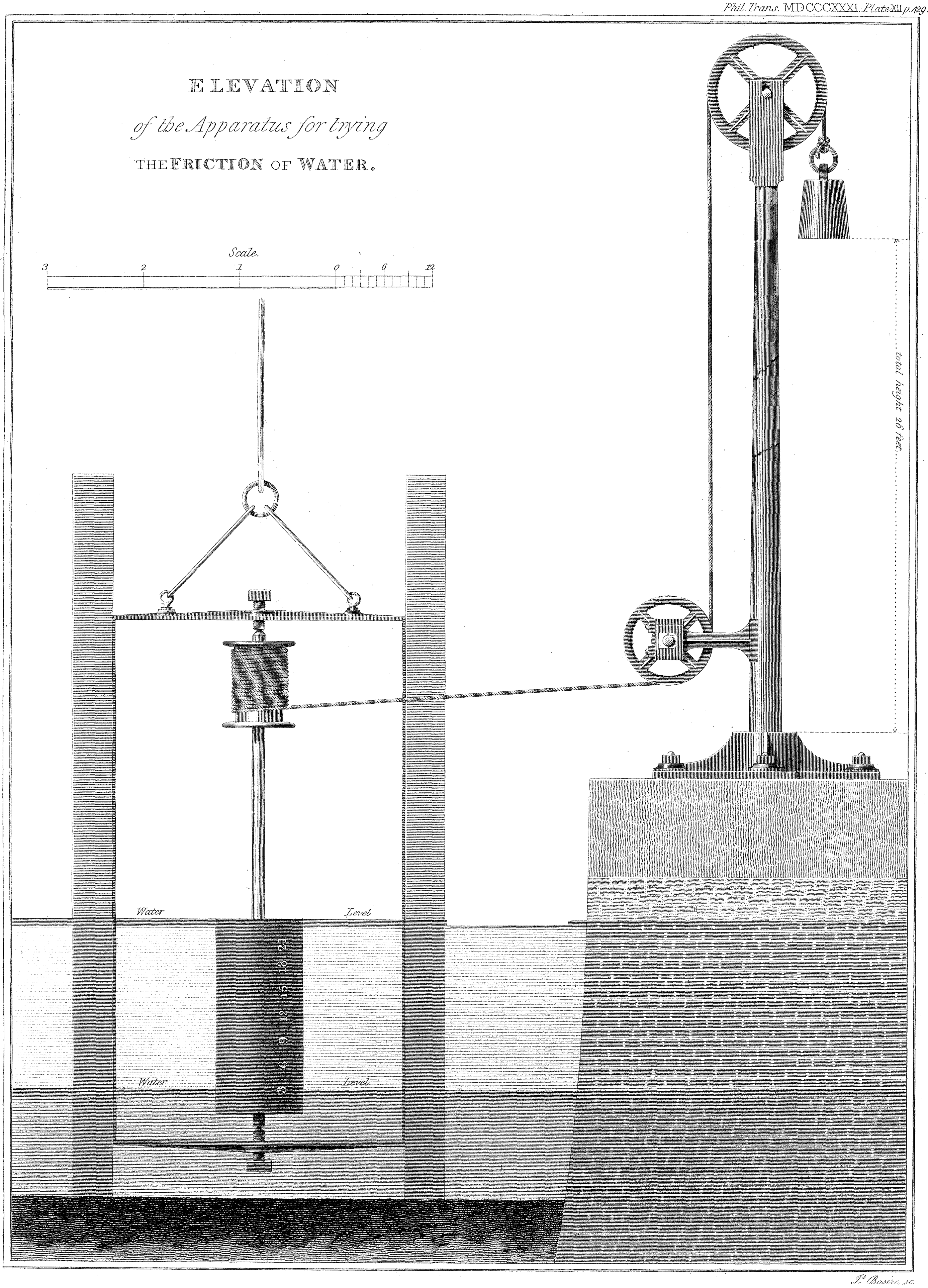
Experiments on the Friction of the Surface of a Cylinder, twenty-four inches long and ten inches three quarters diameter, moving in air and in water.

Table I.

On Surfaces in Water.

Depth of immersion of cylinder.	Weight suspended.	Number of revolutions of cylinder falling the whole height of 26 feet.	Time in descending in water.	Velocity of periphery per second in water.	Time in descending in air.	Velocity of periphery per second in air.	Difference between air and water.	Remarks.
inches.	lbs.		seconds.	inches.	seconds.	inches.	seconds.	
3			15.00	36.021			5.00	
6		Sixteen turns in descending. Periphery moves through 540.32 inches.	18.00	30.017			8.00	Resistance increased by surface with slow velocities, but not in the ratio of the surfaces.
9			25.00	21.612			15.00	
12	1		28.00	19.297	10	54.032	18.00	
15			32.00	16.885			22.00	
18			37.00	14.603			27.00	
21			40.00	13.508			30.00	
24			55.00	9.824			45.00	
3			9.00	60.035			4.00	
6			10.00	54.032			5.00	Resistance scarcely influenced by surface with increased velocities.
9			10.50	51.459			5.50	
12		Ditto.	10.50	51.459	5	108.064	5.50	
15	2		10.50	51.459			5.50	
18			10.50	51.459			5.50	
21			11.00	49.120			6.00	
24			11.00	49.120			6.00	
On Velocities in Water.								
inches.	lbs.		seconds.	inches.	seconds.	inches.	seconds.	
24	4		8.0	67.54	2.45	196.48	5.15	
24	8	Ditto.	6.0	90.053	2.00	270.16	4.00	Could not be tried.
24	16		4.0	135.08	1.50	360.21	2.50	
24	32		2.5	216.128				

ELEVATION
of the Apparatus for trying
THE FRICTION OF WATER.



J. Basire, sc.

Conclusions.

1. That the friction or adhesion of water against the surfaces of solids in motion, approximates the ratio of the surfaces with slow velocities; but that an increase of surface does not materially affect it with increased velocities.

2. That with equal surfaces the velocities do not seem to observe any fixed ratio, but approximate to the squares of the resistance.

With increased velocities the index of the power was found to be less than the duplicate ratio.

To exemplify the result of the foregoing conclusion in a different way,—the cylinder was removed, and circular discs of iron, ten inches and three quarters diameter and one eighth of an inch thick, accurately adjusted to the spindle and polished, were substituted. The friction of the apparatus was again tried, and immersed in the river Thames, as before.

Table II. (See Plate XII. Fig. 2.)

Experiments on the Friction in Water of Circular Discs ten inches and three quarters in diameter and one eighth of an inch thick, revolving with the planes parallel to the horizon, and six inches apart.

Number of discs.	Weight suspended.	Height fallen of weight.	Time of weight descending in water.	Velocity of periphery per second.	Time descending in air.	Velocity of periphery per second in air.	Difference.
lbs.	lbs.		seconds.	inches.	seconds.	inches.	seconds.
	1	Twenty-five feet, mean circle 16.88 would move through 422 inches.	10.00	42.200			8.00
	2		5.00	84.400			3.00
1	3		3.00	140.660	2	211	1.00
	4		3.00	140.660			1.00
	6		3.00	140.660			1.00
	1		15.00	28.133			13.00
	2		6.50	64.923			4.50
2	3	Ditto.	4.50	93.770	2	211	2.50
	4		4.00	105.500			2.00
	6		4.00	105.500			2.00
	1		17.00	24.823			15.00
	2		7.00	60.285			5.00
3	3	Ditto.	5.50	76.727	2	211	3.50
	4		4.00	105.500			2.00
	6		3.00	140.660			1.00
	1		33.00	12.787			31.00
	2		17.00	24.823			15.00
4	3	Ditto.	8.00	52.750	2	211	6.00
	4		6.00	64.923			4.00
	6		4.00	105.500			2.00

Conclusions.

That the friction or adhesion of water is not quite as the surfaces with slow velocities, being in the ratio of one to three instead of one to four, but diminishes rapidly, without observing any ratio in increased velocities*. Hence the resistance of a ship or vessel moving through the water, with an average or higher rate of velocity, forms an inconsiderable portion of the resistance resulting from the displacement of the fluid, and that the brightness observed on the copper of ships after a voyage, may be owing to other causes than the friction of the water simply.

An experiment was made to ascertain the comparative resistance of a pipe revolving in water, and with water running through a pipe; when the resistance was found to be as the surfaces in slow velocities, but to diminish greatly, as before, in high velocities, without observing any fixed ratio.

The above conclusions are in contradiction to those of COULOMB, who did not find that pressure augmented the resistance, but states that the resistance is greater when the immersion is partial.

This apparatus being applicable to fluids generally, advantage was taken of it to ascertain the direct resistance of solids to fluids (see Plate XII.)†, by causing plates and globes to revolve in them, with their planes perpendicular to the plane of the horizon.

As the resistance of solids in fluids does not form the object of this paper, it will be unnecessary to introduce many detailed observations on the subject of these experiments at present, connected as they are with another branch of hydrodynamics. But as it is important to show the relation subsisting between the resistances of cohesion and impulse, I have ventured to detail the following experiments.

* The experiments of the Society for the Improvement of Naval Architecture show a decreased resistance with increased velocities.

† In this case, the number of particles struck will be diminished in the ratio of the radius to the sine of inclination; wherefore the resistance will be diminished in a duplicate ratio of the radius to the sine of inclination. But as the sines of inclination of the two plates are equal, the resistances will be equivalent to the area of one plate (moving perpendicularly to its planes) into the duplicate ratio of the velocity of its motion, and the density of the fluid.

Table III.

Experiments on the Rotations of Iron Discs and Wooden Balls moving in Air, with their planes perpendicular to the plane of the horizon.

Weight suspended.	Height fallen.	Time in descending.					
		Two circular discs $10\frac{3}{4}$ inches diameter. Area 81 inches.	Velocity per second.	Two square fans. Area 81 inches.	Velocity per second.	Two wooden balls $10\frac{3}{4}$ inches diameter.	Velocity per second.
lbs.		seconds.	feet.	seconds.	feet.	seconds.	feet.
2	The spindle made 15.9 turns in falling 25 feet. Mean circle 51.83 would move through 68.67 feet.	10.00	6.867	10.00	6.867	23	2.984
4		6.00	11.445	7.00	9.810	13	5.282
9		4.50	15.261	4.50	15.261	8	8.584
16		3.00	22.891	3.25	21.130	7	9.810
20		2.50	27.469	3.00	22.891	6	11.445

Conclusions.

1. That the resistances are as the squares of the velocity.
2. That the comparative resistances between discs and globes are as two to one nearly.

Table IV.

Experiments on the Resistance of Iron Discs and Wooden Globes revolving in Water.

Weight.	Height fallen.	Time in descending.					
		Two circular discs, 81 inches area.	Velocity per second.	Two square fans, 9 inches square, 81 inches area each.	Velocity per second.	Two wooden balls. Area 81 inches.	Velocity per second.
lbs.		seconds.	feet.	seconds.	feet.	seconds.	feet.
16	The spindle made 15.9 turns in falling 25 feet. Mean circle 51.83 would move through 824.19 inches or 68.67 feet.	63	1.09	53	1.29	15.00	4.57
20		54	1.27	48	1.43	14.00	4.90
32		43	1.59	40	1.71	10.50	6.59
40		40	1.71	35	1.96	9.50	7.22
64		30	2.28	28	2.45	8.00	8.58
256		14	4.90	15	4.57	5.00	13.73

Conclusions.

1. That the resistances are as the square of the velocities.

2. That the mean resistances of circular discs, square plates, and globes in air, are as the numbers 25.180, 22.010, 10.627; and in water, 1.18, 1.36, 0.755; consequently the proportional resistance of air to water, with

Circular discs, is as	1	to	21.3
Plates and fans . .	1	to	16.2
Wooden balls . . .	1	to	2.2

Note.—A portion of the square fans, represented by the letters *a, b, c, d,* in Plate XII. fig. 1, and equal to one fourth of the area of each fan, was cut off, when the resistance was found to be the same as with the square fans.

Experiments on the Quantities of Water discharged by Orifices and Tubes of different diameters and lengths, and at different altitudes.

The phenomena incident to spouting fluids are,

First, The inequality observed in the velocity of the particles comprised in every horizontal section parallel to the orifice.

Secondly, The contraction of the fluid vein beyond the orifice, and consequent diminution of discharge as compared with theory.

Thirdly, The inversion and changes in the sections of the fluid vein at different distances from the orifices.

All these phenomena have been noticed and recorded by various writers, and formulæ adapted to the different circumstances of the expenditure have been given. But neither Bossut nor Du Buat (the most accurate of writers) have recorded a continuous and systematic series of experiments upon the comparative expenditure of orifices and tubes under the circumstances of area, altitude, and length.

The apparatus with which these experiments were performed, consisted of a wooden cistern very accurately made, two feet square inside, and four feet four inches in height. The water was kept at a constant altitude by a regulating cock; and a float having an index attached to it enabled the observer to ascertain the exact height at which the water stood in the cistern above the centre of the orifice.

The orifices were accurately made by Dollond in brass plates one sixtieth of an inch in thickness. The plates were accurately adjusted to a hole in the side of the cistern, and closed by a valve of brass ground to each of the plates. The valve was opened by a lever, and the time noted by chronometers.

The diameters of the tubes, from having been drawn on mandrils, were as accurate as possible; their diameters at the extremities were carefully enlarged, to prevent any wire edges from diminishing their sections; and one extremity of the tube being inserted into a block of hard wood fastened to the cistern, and the other stopped by a valve, the experiments were recorded as before.

Table V.

Experiments on the Quantity of Water discharged by different-sized Orifices from a vessel kept constantly full and at different heights.

Circular Orifice made in a brass plate 1 inch diameter, $\frac{1}{60}$ inch thick.				
Constant height of the surface of the water above the centre of the orifice.	Real time in discharging one cubic foot.	Theoretical time in discharging one cubic foot, $t = \frac{Q}{2A\sqrt{gH}}$.	Ratio of the theoretical to the real discharges.	Vena contracta.
feet.	seconds.	seconds.		
4	19.50	11.4	1 : .584	Not accurately measured.
3	21	13.2	1 : .628	
2	26	16.1	1 : .619	
1	36	22.8	1 : .633	
Circular Orifice in a brass plate $\frac{3}{4}$ inch diameter, $\frac{1}{60}$th inch thick.				
4	33	20.3	1 : .614	At six tenths of an inch from the orifice, the diameter had contracted to 0.685 of an inch.
3	37	23.4	1 : .632	
2	44	28.7	1 : .652	
1	63	40.6	1 : .644	
Circular Orifice in a brass plate $\frac{1}{2}$ inch diameter, $\frac{1}{60}$th inch thick.				
4	73	45.7	1 : .626	At half an inch beyond the orifice, the diameter contracted to 0.37 of an inch.
3	83	52.8	1 : .636	
2	104	64.6	1 : .621	
1	144	91.4	1 : .634	
Circular Orifice in a brass plate $\frac{1}{4}$ inch diameter, $\frac{1}{60}$th inch thick.				
4	276	182.9	1 : .662	At a quarter of an inch beyond the orifice, the diameter contracted to one twentieth of an inch less than the orifice.
3	320	211.3	1 : .660	
2	396	258.6	1 : .653	
1	545	365.7	1 : .671	

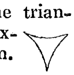
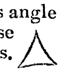
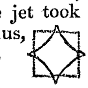
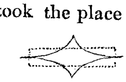
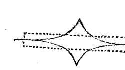
N.B. Each result shows the mean of four experiments.

Remarks.

The phenomena relative to the form and direction of veins of spouting fluids, and the remarkable inversion of the fluid veins at certain distances from their orifices, have been so fully noticed in "Experiences sur la Forme et sur la Direction des Veines et des Courans d'Eau; par George Bidone: Turin, 1829," that it is unnecessary to state further than that they have been completely corroborated in the foregoing experiments.

Table VI.

Experiments on the Quantities of Water discharged from Rectangular and Triangular Orifices in brass plates one sixtieth of an inch thick, and of equal areas, from a vessel kept constantly full and at different heights.

Constant height of the surface of the water above the centre of the orifice.	Time in discharging one cubic foot.	Theoretical time in discharging one cubic foot, $\frac{Q}{2A\sqrt{g}H}$.	Ratio of real to theoretical discharge.	Form of orifice.
Equilateral Triangle whose area is one inch, and angle uppermost.				
feet.	seconds.	seconds.		
4 3 2 1	15 18 22 30	8.9 10.3 12.7 17.9	1 : .593 1 : .572 1 : .577 1 : .596	Vena contracta about half an inch beyond the orifice; but the jet with the angles reversed, and taking the sides of the triangle, the jet afterwards expanded and lost its form. 
Equilateral Triangle as before, with the angle downwards.				
4	15	8.9	1 : .593	Vena contracta the same as before, but the jet having its angle upwards, being the reverse of the former experiments. 
Rectangular Orifice of one square inch.				
4 3 2 1	15 17 20 29	8.9 10.3 12.7 17.9	1 : .593 1 : .606 1 : .635 1 : .617	Vena contracta about three quarters of an inch beyond the orifice, when each angle of the jet took the place of a side thus, and dissipated in spray. 
Rectangular Orifice 2 inches long, ½ an inch wide, having the long side parallel to the surface of the water.				
4 3 2 1	15 17 20 29	8.9 10.3 12.7 17.9	1 : .593 1 : .606 1 : .635 1 : .617	Vena contracta as before. Each angle of the jet took the place of a side. 
Rectangular Jet 1½ inch long, ⅝ wide, placed as before.				
4 3 2 1	15 17 19 27	8.9 10.3 12.7 17.9	1 : .593 1 : .606 1 : .668 1 : .663	Vena contracta as before. 

Remarks.

That with equal areas, the expenditure by different orifices, whether circular, rectangular, or triangular, is nearly the same, the increase being in favour of rectangular orifices.

TABLE VII.

Experiments on the Quantity of Water discharged by Cylindrical Glass Orifices and Tubes, from one inch in length to one foot, and of different diameters, from a vessel kept constantly full, and at different heights.

Constant height of the surface of water above the centre of orifice.	Time in seconds in discharging one cubic foot.				Remarks.
	1 inch.	$\frac{3}{4}$ in.	$\frac{1}{2}$ in.	$\frac{1}{4}$ in.	
feet.					
4	11.5	24.5	55	145	In comparing these experiments with the time and quantity discharged by plate orifices, there is a diminution of time, and an increased discharge of from $\frac{1}{5}$ to $\frac{1}{4}$.
3	15.0	28.5	63	157	
2	17.5	35.0	77	205	
1	25.0	53.0	110	297	
From Glass Tubes one foot long.					
	1 inch.	$\frac{3}{4}$ in.	$\frac{1}{2}$ in.	$\frac{1}{4}$ in.	
4	14.0	30	63	200	Shows an increase of time and a diminution of discharge in the ratio of from $\frac{1}{9}$ to $\frac{1}{4}$.
3	17.0	33	73	227	
2	21.5	40	88	283	
1	30.0	58	130	410	

Conclusions.

1. That the quantities discharged in equal times by orifices and additional tubes, are as the areas of the orifices.

2. That the quantities discharged in equal times by the same additional tubes and orifices under different heads, are nearly as the square roots of the corresponding heights.

3. That the quantities discharged in equal times by the different additional tubes and orifices under different heights, are to one another in the compound ratio of the areas of the apertures, and of the square roots of the heights.

From the foregoing experiments the mean coefficient for altitudes of 4 feet with the circular orifices,

is 0.621
but with altitudes of 1 foot the coefficient is 0.645
with triangular orifices at 4 feet altitude 0.593
with triangular orifices at 1 foot altitude 0.596
with rectangular orifices at 4 feet altitude 0.593
with rectangular orifices at 1 foot altitude 0.616

Hence, allowing for the inaccuracies incident to experiments of this nature, we may safely adopt Messrs. PRONY and BOSSUT's coefficients for altitudes of 4 feet 0.621
—— —— —— 1 foot 0.619

In the case of additional tubes of glass the coefficient is much higher than BOSSUT's, which was for 4 feet 0.806, and 1 foot 0.817.

Note.—Vide VENTURI and EYTELWEIN's experiments.

Let A = area of orifice in square feet.

d = diameter of orifice if circular.

H = altitude of the fluid in feet.

T = time.

g = gravity in one second.

Q = quantity of water in cubic feet.

According to BOSSUT's experiments $Q = 0.61938\,A\,T\sqrt{2\,g\,H}$.

And as $2\,g$ is a constant quantity, and is equal to 7.77125, we have $Q = 4.818\,A\,T\sqrt{H}$ for orifices of any form, substituting d if circular, or $Q = 3.7842\,d^2\,T\sqrt{H}$.

From the second of these equations we obtain

$$A = \frac{Q}{4.818\,T\sqrt{H}}\quad T = \frac{Q}{4.818\,A\sqrt{H}}\quad \text{and } H = \frac{Q}{(4.818\,A\,T)^2}.$$

For additional tubes the equation will stand thus: $Q = 0.81\,A\,T\sqrt{2\,g\,H}$; but since $2\,g$ is constant, and is 7.77125, we have $Q = 4.9438\,d^2\,T\sqrt{H}$, from which we deduce

$$d = \sqrt{\frac{Q}{4.9438\,T\sqrt{H}}}\quad T = \frac{Q}{4.9438\,d^2\sqrt{H}}\quad H = \frac{Q}{(4.9438\,d^2\,T)}^2.$$

TABLE VIII.

Experiments on the Friction or Quantity of Water discharged by Leaden Pipes of different diameters and lengths, from a vessel kept constantly full, and at different heights.

Pipes 15 feet long each, straight.					
Constant height of the surface of the water above the centre of the pipe.	Time in discharging one cubic foot.				Remarks.
	1 inch.	$\frac{3}{4}$ in.	$\frac{1}{2}$ in.	No leaden pipes to be had $\frac{1}{4}$ bore.	
feet.	seconds.	seconds.	seconds.		
4	28	54	143		The time in discharging one cubic foot is nearly double the time occupied by glass tubes of equal lengths and areas.
3	33	63	164		
2	$41\frac{1}{4}$	79	208		
1	$61\frac{1}{4}$	117	312		

Table IX.

Experiments on the Quantities of Water discharged by Leaden Pipes $\frac{1}{2}$ inch bore, but of different lengths from one foot to thirty feet in length.

Glass tubes 1 inch long, $\frac{1}{2}$ inch diam.		Brass orifice $\frac{1}{2}$ diam.	1 foot long.	3 ft. 9 in.	7 ft. 6 in.	11 ft. 3 in.	15 ft.	30 ft.	Remarks.
feet.	seconds.	seconds.	seconds.	seconds.	seconds.	seconds.	seconds.	seconds.	
4	55	73	55	78	102	122	143	203	The ratio of discharge by glass tubes with pipes of 30 feet long, is as 1 : 4 } nearly.
3	63	83	63	92	120	145	164	240	Ditto with brass orifices, is as 1 : 3 } nearly.
2	77	104	93	113	151	184	208	303	
1	110	144	133	170	226	278	312	450	

Conclusions on Pipes of different lengths.

That the expenditures of water by pipes of equal diameters but of unequal lengths and under different altitudes, are nearly as follow.

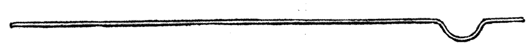
The length being as 30 to 1, the expenditures are as 3.7 to 1
Do. . . . 8 to 1 . . . do. . . . 2.6 to 1
Do. . . . 4 to 1 . . . do. . . . 2.0 to 1
Do. . . . 2 to 1 . . . do. . . . 1.4 to 1

The discharges by glass and leaden tubes are nearly alike. The length of a pipe may be increased from 3 to 4 feet without diminishing the discharge as compared with the plate orifices.

TABLE X.

Experiments on Leaden Pipes with Flexures.

The straight pipe of $\frac{1}{2}$ an inch bore, on which the preceding experiments were made, was carefully bent into one, two, and fourteen semicircular bends respectively, each of $7\frac{1}{2}$ inches in the semidiameter, and two of $\frac{1}{4}$th part of a circle of $3\frac{1}{4}$ inches radius. One end of the pipe was fixed in the wooden orifice as before, and the following are the results.



Pipe 15 feet long, $\frac{1}{2}$ inch bore, with one semicircular and two $\frac{1}{4}$-circle bends.

Constant height of the surface of the water above the centre of the orifice.	Time in discharging one cubic foot by a pipe with three bends.	Time discharging one cubic foot by a straight pipe.	Remarks.
feet.	seconds.	seconds.	
4	147	143	The position of the bends, whether vertical or horizontal, at either extremity of the pipe, does not affect the result.
3	175	164	
2	213	208	
1	316	312	

Pipe 15 feet long, $\frac{1}{2}$ inch bore, with 14 semicircular and two $\frac{1}{4}$-circle bends.

feet.	seconds.	seconds.	
4	162	143	The expenditure is diminished by the bends from $\frac{1}{5}$ to $\frac{1}{3}$, which represents the friction of the pipe.
3	200	164	
2	247	208	
1	351	312	

Results.

1. That with one semicircular and two $\frac{1}{4}$ of a circle bends, as compared with a straight pipe of equal length and bore, the resistance varies from $\frac{1}{36}$th to $\frac{1}{70}$th part of the resistance of the straight pipe.

2. That with fourteen semicircular and two quarter of a circle bends, the resistance varies from $\frac{1}{19}$th to $\frac{1}{39}$th of the resistance of a straight pipe.

3. That the increased number of bends does not increase the resistance in the ratio of the number of bends, but merely shows an increased resistance, as compared with the four bends, of $\frac{1}{15}$th to $\frac{1}{35}$th.

TABLE XI.

Experiments on the Discharge of Water by Leaden Pipes of $\frac{1}{2}$ an inch bore, 15 feet long, but bent in the forms of from one to twenty-four right-angled elbows, each side being $6\frac{3}{4}$ inches long.

Height of the surface of the water above the centre of the orifice.	One right angle $8\frac{1}{2}$ inches from the end of the pipe.	Straight pipe 15 feet long.	Twenty-four right angles.	Remarks.
feet.	seconds.	seconds.	seconds.	
4	180	143	395	In the first three experiments we have a diminution of expenditure in the ratio of $2\frac{1}{2}$ to 1, and in the last experiment as 3 to 1 nearly.
3	214	164	465	
2	246	208	584	
1	371	312	872	

Conclusions.

From the foregoing experiments with one rectangular pipe, it would be reasonable to conclude that the diminution of discharge would be as the number of right angles; but comparing the expenditure by one right-angled pipe with the expenditure of a pipe with twenty-four right angles, the difference is only in the ratio of about two to one.

General Remarks on the Expenditure of Horizontal and Bent Pipes.

Formulæ adapted to the different circumstances of the motion of water in pipes and conduits have been given by various authors.

By some, the retardations were supposed to be in the inverse ratios of the squares of the lengths of the pipes; and by others, to be represented by a certain portion of the altitude of the reservoir above the centre of the pipe, the resistance being directly as the length and circumference of the pipe, and inversely as the area of the section.

M. GIRARD, in his beautiful experiments*, conceived the resistance to be compounded of the first and second powers of the velocity. So that, deducing the values from DUBUAT's experiments, and expressing the resistance due to cohesion by $R x U$, R being the quantity to be obtained by experiment, and making

* Memoires des Scavans Etrangers.

the resistance due to the asperities equal to $R x U^2$, the sum of the resistance is $R (U + U)^2$.

M. Prony, applying his profound acquirements to the solution of all the cases of preceding authors, deduced from a selection of upwards of fifty experiments the following simple formula: $U = 26.79 \sqrt{\frac{DZ}{\lambda}}$;

U being the mean velocity of the section of the pipe;
D the diameter of the pipe;
Z the altitude of the water;
λ the length of the pipe:

from which it appears that the velocity is directly in the compound ratio of the square roots of the diameter of the pipe and head of water, and inversely as the square roots of the length of the pipe; that is, for any given head of water and diameter of pipe, the velocity is inversely as the square root of the length of the pipe.

If we compare these results with those of Dubuat, Girard, and others, they approximate very nearly to each other.

In general, if we incline a pipe to an angle of about 6½ degrees, or one ninth of its length, the discharge will be nearly equal to the discharge by additional tubes. The charge necessary to express the mean velocity of water issuing from straight pipes is by some authors equal to $\frac{V^2}{478}$ *; Dr. Young makes it $\frac{V^2}{550}$: the diminution of expenditure depending upon the contraction of the fluid vein and the friction of the pipe.

The change occasioned by bends and angles in the direction of the fluid vein tends to diminish the velocity in a very remarkable manner.

Dubuat undertook several experiments upon this subject, but the formula proposed by him does not solve the difficulty, where $\frac{V^2 S^2}{m}$ gives the resistance due to one bend, V being the velocity, S the sine of incidence or reflection, and m a constant quantity determined by Dubuat to be 2998.50.

Now although it is reasonable to suppose that the resistance should be proportionable to the squares of the sines of the angles of incidence, yet as all

* Dubuat and Langsdorf.

the particles of the fluid vein are not reflected in the same angle, and as a considerable portion of the velocity is destroyed by the first angle or bend the fluid meets with in the pipe, M. DUBUAT'S theory is fundamentally erroneous, the more especially as he has rejected more than one half of the twenty-five experiments mentioned by him. Dr. YOUNG'S suppositions, of the resistance being as the angular flexure and the power of the radius, of which the index is $\frac{7}{8}$, are equally erroneous, as is evinced by the foregoing experiments.

In conclusion, it is evident that the subject of friction admits of an immense variety of applications. To determine the measure of the resistances experienced by vessels and floating bodies in their motion through fluids ; the law of the retardations of rivers, and the cause of the obstructions presented to the waves of the ocean in the slopes assumed by its shores; the equilibrium of earths, and their connection with solids and fluids,—all of them are questions of the utmost importance in the economy of nature, and their solution can only be attained by an accumulation of facts.

N.B. Since the foregoing paper was presented to the Royal Society, an abstract of an extensive series of experiments on the expenditure of water through rectangular orifices of large dimensions, has been submitted to the French Academy by Messrs. PONCELET and LESBROS, of the Corps de Genie at Metz ; and as these experiments were undertaken by order of the French government, no expense was spared to have them made as extensive as possible. Their objects were principally to ascertain the exact measure of the coefficient of contraction and the forms of the fluid veins under different altitudes and areas.

The results of which are :—

That with an orifice of 20 centimetres square, the coefficient is 0.600 under altitudes of 1 metre 68 centimetres. But when the altitude was reduced to four or five times the opening of the orifice, the coefficient increased to 0.605, but again diminished rapidly as the altitude diminished, to 0.593.

That with orifices of smaller dimensions, i. e. from 10 to 5 centimetres square, the same law was observed, the coefficients being respectively 0.611, 0.618, and 0.611, for opening of 10 and for 5 centimetres, 0.618, 0.631, 0.623 ; and for orifices of less dimension, the coefficient continually increased up to 0.698.

That for water running over weirs, the mean coefficient was 0.400, which differs very little from that of BIDONE.

Hence we see little reason to deviate from the coefficients already given.